Manipulation and control of atomic motion by light forces

Leandro Alvares Machado

Institute of physics of São Carlos
University of São Paulo

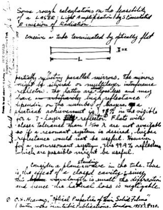
July 7, 2022

UNIVERSIDADE DE SÃO PAULO
Instituto de Física de São Carlos

Table of Contents

The invention of Laser

- **1957**: Gordon Gould coined the acronym LASER and described the essential elements for constructing one.
- **1960**: Theodore H. Maiman constructs the first laser using a cylinder of synthetic ruby



Historical context

1993 - Steven Chu, Claude Cohen-Tannoudji e William Phillips

- Atomic cooling with light
- Optical and magnetic trapping
- Theoretical frameworks about light matter interaction
- Evaporative cooling

(1997)

Optical forces

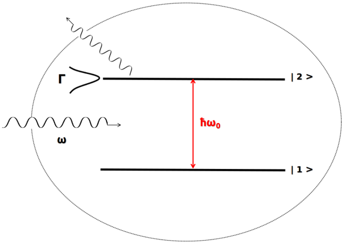
Assumptions for the semiclassical model

- A two-level atom
- Interaction with a classical radiation field with frequency ω
- There is dissipation by spontaneous emission with rate Γ
- Interest only in the internal timescale of the system
- Dipole approximation ($a_0 << \lambda$)

Optical forces

The two-level atom under perturbation of a classical radiation field

$$\mathbf{E} = \frac{\epsilon}{2}\left(E_0 e^{i(\mathbf{k}.\mathbf{r}-\omega t)} + E_0^* e^{-i(\mathbf{k}.\mathbf{r}-\omega t)}\right)$$



Non-perturbed Hamiltonian

$$\hat{H}_0 |1\rangle = E_1 |1\rangle$$
$$\hat{H}_0 |2\rangle = E_2 |2\rangle$$

$$\Longrightarrow \omega_0 = \frac{(E_2 - E_1)}{\hbar}$$

Optical forces

Under a perturbative treatment:

$$\hat{H} = \hat{H}_0 + \hat{H}'(t) \ , \tag{1}$$

with $\hat{H}'(t) = -\hat{\mathbf{d}}.\mathbf{E}$.
In matrix notation

$$\hat{H} = \hbar\omega_0 \left|2\right\rangle \left\langle 2\right| + \mathbf{d}.\mathbf{E} \left|2\right\rangle \left\langle 1\right| + \mathbf{d}^*.\mathbf{E} \left|1\right\rangle \left\langle 2\right| \tag{2}$$

$$= \begin{pmatrix} 0 & \frac{\hbar\Omega^*(\mathbf{r})}{2} e^{-i(\mathbf{k}.\mathbf{r}-\omega t)} \\ \frac{\hbar\Omega(\mathbf{r})}{2} e^{i(\mathbf{k}.\mathbf{r}-\omega t)} & \hbar\omega_0 \end{pmatrix} \tag{3}$$

where $\Omega(\mathbf{r}) \equiv \left\langle 1 \right| e\hat{\mathbf{r}} E_0 \boldsymbol{\epsilon} \left| 2 \right\rangle / \hbar$ and we neglect terms rotating like $(\omega + \omega_0)$ (RWA).

Optical forces

In the interaction picture and applying an unitary transform to pass our system to the rotating frame

$$\hat{U} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\Delta t} \end{pmatrix} . \tag{4}$$

with $\Delta = \omega - \omega_0$. So, evaluating the Hamiltonian by $H_{RF} = U^\dagger H U + i\hbar \dot{U}^\dagger U$ and writing the Rabi frequency as $\Omega = |\Omega| e^{i\theta}$

$$\hat{H}_{RF} = \begin{pmatrix} 0 & \frac{\hbar\Omega}{2} \\ \frac{\hbar\Omega}{2} & -\hbar\Delta \end{pmatrix} . \tag{5}$$

Introducing the spontaneous emission, we have to make use of the density operator formalism $\hat{\rho} = |\psi\rangle\langle\psi|$. The time evolution is given by the Liouville equation

$$\frac{d\hat{\rho}(t)}{dt} = \frac{i}{\hbar}[\hat{\rho}(t), \hat{H}_{RF}] \ . \tag{6}$$

Defining the Bloch vector as

$$\boldsymbol{\rho} \equiv \begin{pmatrix} \rho_{11} \\ \rho_{22} \\ \rho_{12} \\ \rho_{21} \end{pmatrix} \ , \tag{7}$$

the time evolution is usually rewrites in the form of so-called Bloch equations

$$\dot{\boldsymbol{\rho}} = \mathcal{M}\boldsymbol{\rho} \ , \tag{8}$$

where $\mathcal{M}$ is the transition matrix.

Optical forces

To introduce the dissipation by spontaneous emissiom, we replace

$$\frac{d}{dt} \rightarrow \left(\frac{d}{dt} + \gamma \right) \ . \tag{9}$$

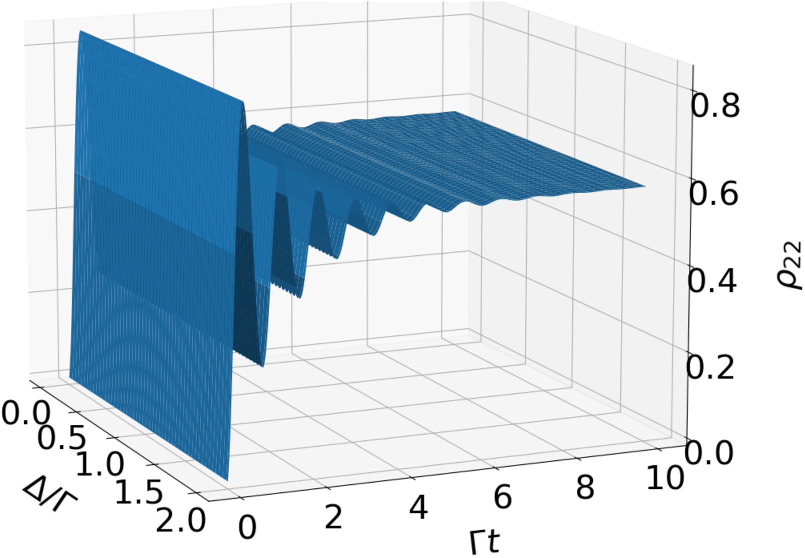
Therefore, for a two-level system,

$$\mathcal{M} = \begin{pmatrix} 0 & \Gamma & \frac{i}{2}\Omega & -\frac{i}{2}\Omega \\ 0 & -\Gamma & -\frac{i}{2}\Omega & \frac{i}{2}\Omega \\ \frac{i}{2}\Omega & -\frac{i}{2}\Omega & -i\Delta - \gamma & 0 \\ -\frac{i}{2}\Omega & \frac{i}{2}\Omega & 0 & i\Delta - \gamma \end{pmatrix} \ , \tag{10}$$

being Γ the spontaneous emission rate and $\gamma = \Gamma/2$.

The dissipation, after a long time, exponentially brings the system to a steady state.

Optical forces



Optical forces

To find the steady state, we should to solve $\dot{\rho}(t \to \infty) = 0$.
we obtain the population

$$\rho_{22} = \frac{\frac{1}{4}|\Omega|^2}{\Delta^2 + \frac{1}{2}|\Omega|^2 + \frac{1}{4}\Gamma^2} \qquad (11)$$

and the coherence

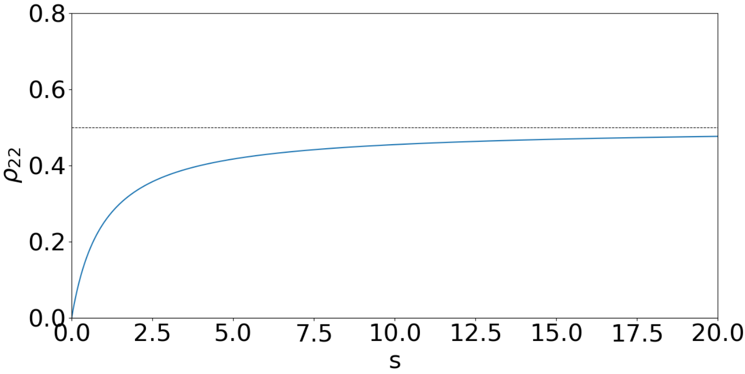
$$\rho_{12} = e^{i\Delta t}\frac{\frac{1}{2}\Omega(\Delta - \frac{i}{2}\Gamma)}{\Delta^2 + \frac{1}{2}|\Omega|^2 + \frac{1}{4}\Gamma^2} \; . \qquad (12)$$

Note the Lorentzian frequency dependence. It means that the system has a signature of the saturation broadening effect.

$$s \equiv \frac{2|\Omega|^2}{4\Delta^2 + \Gamma^2} \; . \qquad (13)$$

Rewriting the excited state population and the coherence in terms of saturation parameter

$$\boxed{\rho_{22} = \frac{s/2}{1+s} \quad , \quad \rho_{12} = e^{i\Delta t}\frac{\Delta - i\Gamma/2}{\Omega}\frac{s}{1+s}} \tag{14}$$



Optical forces

To compute the optical forces, we make use of Ehrenfest theorem

$$\mathbf{F}(\mathbf{r}) = \left\langle \hat{\mathbf{F}}(\mathbf{r}) \right\rangle = \frac{d \left\langle \hat{\mathbf{p}} \right\rangle}{dt} \,. \tag{15}$$

By using the Heisenberg picture $\frac{d\langle \hat{A} \rangle}{dt} = \frac{i}{\hbar} \left\langle [\hat{H}, \hat{A}] \right\rangle$ and remembering that $[\hat{H}, \hat{\mathbf{p}}] = i\hbar \nabla \hat{H}$

$$\mathbf{F}(\mathbf{r}) = \left\langle \hat{\mathbf{F}}(\mathbf{r}) \right\rangle = -Tr(\hat{\rho} \nabla \hat{H}) \,. \tag{16}$$

Evaluating at $\mathbf{r} = 0$, for simplicity

$$\begin{aligned} \mathbf{F} &= -\frac{\hbar \Delta}{6} \frac{1}{1+s} \nabla s + \frac{\hbar \mathbf{k} \Gamma}{6} \frac{s}{s+1} \\ &= -\frac{\hbar \Delta}{6} \nabla \ln\left[1+s\right] + \frac{\hbar \mathbf{k} \Gamma}{6} \frac{s}{s+1} \end{aligned} \,. \tag{17}$$

UNIVERSIDADE DE SÃO PAULO
Instituto de Física de São Carlos

Optical forces

The light force comprises **two contributions**: the first one is the dipolar force, in which is given by

$$\mathbf{F}_{dp} = -\frac{\hbar\Delta}{6}\boldsymbol{\nabla}\ln\left[1+s\right] \ . \quad (18)$$

The second one is the radiation pressure given by

$$\mathbf{F}_{rp} = \frac{\hbar\mathbf{k}\Gamma}{6}\frac{s}{s+1}. \quad (19)$$

UNIVERSIDADE DE SÃO PAULO
Instituto de Física de São Carlos

Characteristics of dipolar force

$$\mathbf{F}_{dp} = -\frac{\hbar\Delta}{6}\boldsymbol{\nabla}\ln\left[1+s\right]$$

Optical Forces

Characteristics of dipolar force:

- Can be written as $\mathbf{F}_{dp} = -\nabla U_{dp}$, where $U_{dp} = \frac{\hbar\Delta}{6} \ln[1+s]$

$$\Longrightarrow \text{ A conservative force!}$$

Stimulated process without energy exchange between the field and the atom.

Optical Forces

- Control the strength of the dipolar force by focusing the light

The force gradient increases with increasing strength with no upper limit.

- Control the signal of dipolar force by tuning the optical frequency

$$\text{When } \Delta > 0 \implies \text{dispersive force!}$$

$$\text{When } \Delta < 0 \implies \text{attractive force!}$$

Characteristics of radiation pressure

$$\mathbf{F}_{rp} = \frac{\hbar \mathbf{k} \Gamma}{6} \frac{s}{s+1}$$

Characteristics of radiation pressure:

- Dissipative force

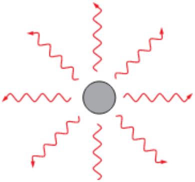
> Spontaneous force arises from the recoil experienced by atom when it absorbs or emits a quantum of light $\hbar\mathbf{k}$

$$\mathbf{F}_{rp} = \underbrace{\hbar\mathbf{k}}_{photon\ momentum} \times \underbrace{\frac{\Gamma}{6}\frac{s}{s+1}}_{effective\ absorption\ rate}$$

Optical Forces

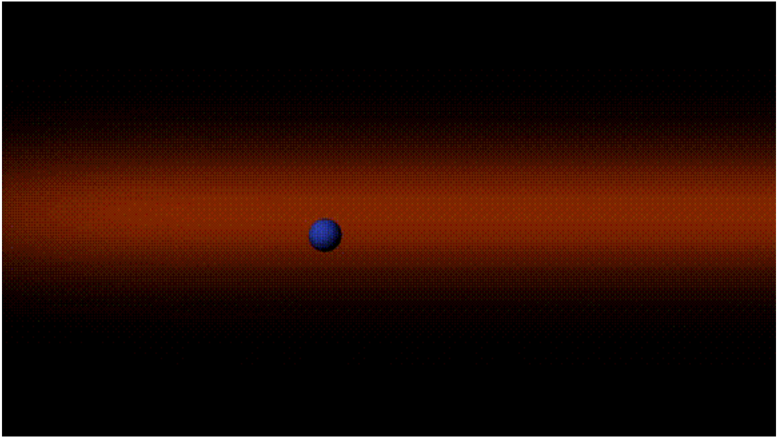
- The effective absorption rate define an **upper limit** to its magnitude and therefore define a **minimum temperature**
- Each absorbed photon transfers $\hbar\mathbf{k}$ in the **direction of propagation**

- The spontaneous emission occurs in **random directions** (isotropic)



Optical Forces

- When **averaged over many cycles**, the atom undergoes a **recoil**.

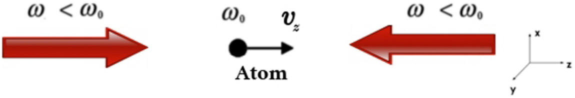


Main applications

- Cooling atoms: The optical molasses technique
- Trapping atoms: The dipolar trap

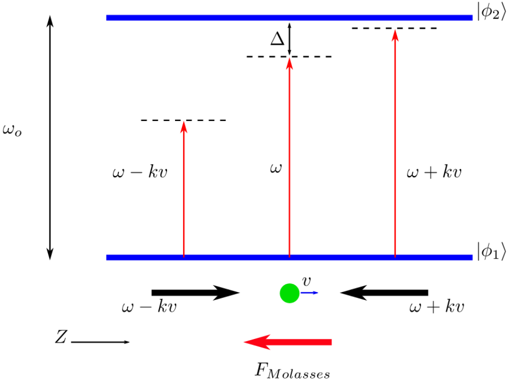
$1D$ model:

- An atom propagates in the z direction with velocity v_z
- 2 laser beams detuned by Δ propagating in $\pm z$



Cooling atoms: The optical molasses technique

- Due to the Doppler $\Delta \to \Delta \pm kv_z$, where kv_z is the Doppler shift



Now, it's convenient rewrites the saturation parameter as $s = \frac{\Omega}{\Gamma^2/2}$

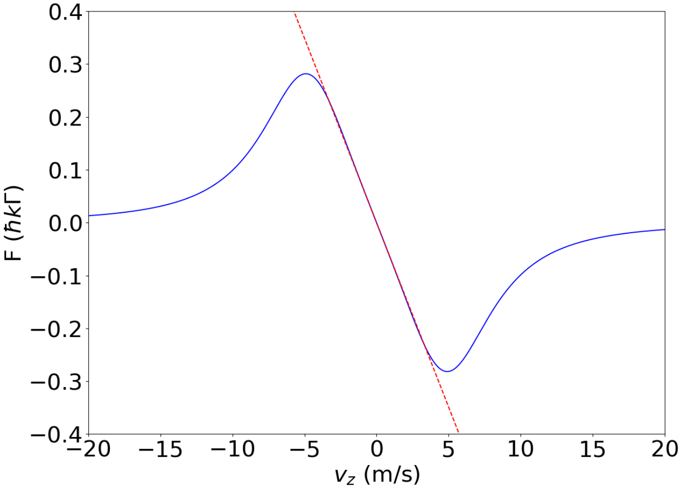
$$\implies \mathbf{F_{rp}} = \frac{\hbar \mathbf{k} \Gamma}{2} \frac{s}{(2\Delta/\Gamma)^2 + 1 + s} \ . \tag{20}$$

The total force acting on atom is $\mathbf{F} = \mathbf{F}_+ + \mathbf{F}_-$, where

$$\mathbf{F}_\pm = \pm \frac{\hbar \mathbf{k} \Gamma}{2} \frac{s}{(2(\Delta \mp k v_z)/\Gamma)^2 + 1 + s} \tag{21}$$

Cooling atoms: The optical molasses technique

$$\mathbf{F}_{\pm} = \pm \frac{\hbar \mathbf{k} \Gamma}{2} \frac{s}{(2(\Delta \mp k v_z)/\Gamma)^2 + 1 + s}$$



For $kv_z << \Omega$ and Γ, expanding in Taylor series

$$F_z \approx 4\hbar ks \frac{kv_z(2\Delta/\Gamma)}{[1+s+(2\Delta/\Gamma)^2]^2} \ . \tag{22}$$

Note that, for $\Delta < 0$ (red side of resonance)

$$\boxed{\mathrm{F}_z \approx -\alpha v_z \quad with \quad \alpha = s\frac{4\hbar k^2(2\Delta/\Gamma)}{[1+s+(2\Delta/\Gamma)^2]^2}} \tag{23}$$

- $\Longrightarrow$ Dissipative/viscous force

The momentum fluctuations in the absorption-emission process will generate a **heating rate** that, at some point, will be balanced with the cooling rate

$$\left(\frac{dE}{dt}\right)_{heat} = -\left(\frac{dE}{dt}\right)_{cool} , \tag{24}$$

However,

$$\left(\frac{dE}{dt}\right)_{cool} = Fv \approx -\alpha v^2$$

$$\left(\frac{dE}{dt}\right)_{heat} = \frac{d}{dt}\frac{\langle \hat{p}^2 \rangle}{2m} = \frac{\hbar^2 k^2}{2m} 2R$$

being $R = \frac{F_+ + F_-}{\hbar k}$ the scattering rate.

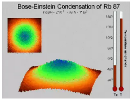
Cooling atoms: The optical molasses technique

Thus, the balance equation become

$$k_B T = mv^2 = \frac{m\left(\frac{dE}{dt}\right)}{\alpha} \approx \frac{\hbar\Gamma}{4}\left(\frac{2\Delta}{\Gamma} + \frac{\Gamma}{2\Delta}\right) . \qquad (25)$$

Minimum when $\Delta = \Gamma/2$

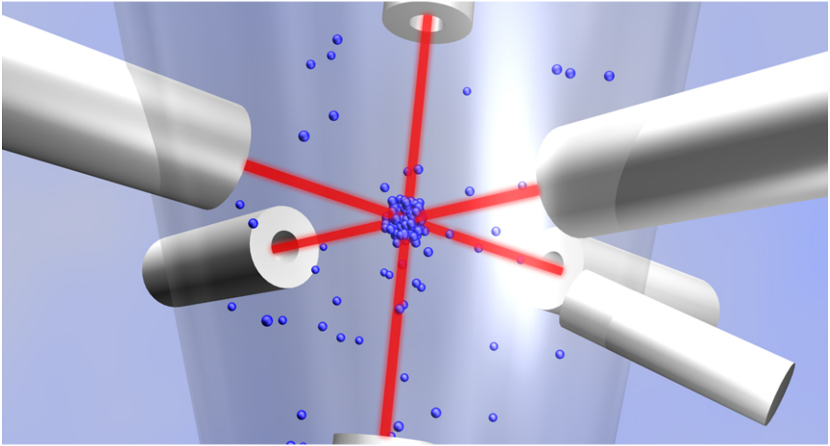
$$\boxed{T_{min} \approx \frac{\hbar}{2k_B}\Gamma} \qquad (26)$$



For Alkaline atoms $T_{min} \approx 10^1\ \mu K$. Others cooling techniques is required to achieve the Bose-Einstein condensation ($\approx 10^2\ nK$).

Cooling atoms: The optical molasses technique

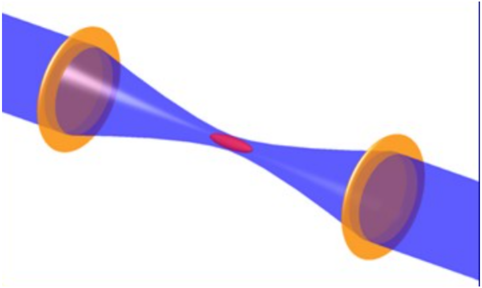
To optimize the optical cooling in 3D case



Trapping atoms: The dipole trap

Using the dipolar force, we can trap the atoms by setting up our laser beam

- Tuning optical frequency far from resonance
- $\Delta < 0$
- Strong field gradient



$$\mathbf{F}_{dp} \approx -\boldsymbol{\nabla} \frac{\hbar\Omega(\mathbf{r})^2}{4\Delta} \qquad (27)$$

Trapping atoms: The dipole trap

Rewriting the forces in terms of the intensity

$$U_{dp}(\mathbf{r}) \propto \frac{\Gamma}{\Delta} I(\mathbf{r}) \quad \textit{and} \quad \mathbf{F}_{rp} \propto \left(\frac{\Gamma}{\Delta}\right)^2 I(\mathbf{r}) \tag{28}$$

For very high intensity and large detuning:

- trapping is maintained
- atoms do not absorbs photon

Trapping atoms: The dipole trap

Considering an usual Gaussian beam

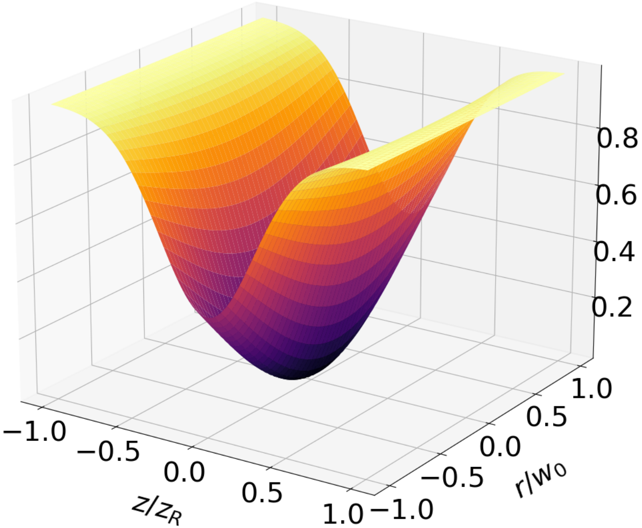
$$I(\mathbf{r}) = \frac{2P}{\pi w_0^2} e^{(-2x^2-2y^2)/w_0^2} e^{-z^2/z_R^2} \ , \tag{29}$$

P is the total power of the beam and $Z_R \equiv \pi w_0^2/\lambda$ the *Rayleigh length*.

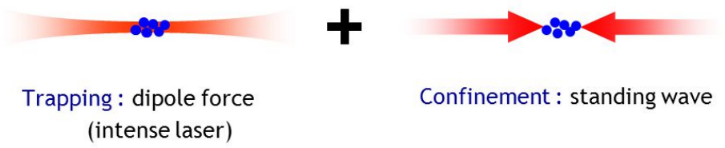
- red-detuned light ($\Delta < 0$)
- near the center ($r << w_0/2$)
- Rayleigh length ($z << \pi w_0^2/\lambda$)

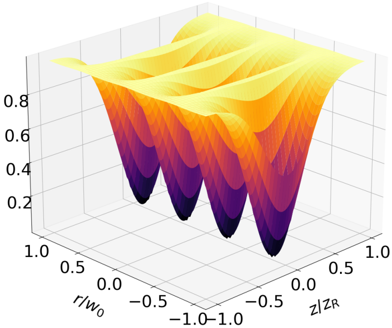
$$\Longrightarrow \ U(\mathbf{r}) \propto c_1 \left(c_2 + \frac{r^2}{w_0^2(2c_2)^{-1}} + \frac{z^2}{z_R^2(2c_2)^{-1}} \right) \tag{30}$$

Trapping atoms: The dipole trap



Trapping atoms: The dipole trap





Conclusion

- Dipolar Force has a conservative character
- Radiation pressure is a dissipative force based on momentum transfer
- Allowed revolucionary applications such as optical cooling, dipolar traps, MOT...
- The experimentals techniques allowed the realization of many other experiments (BEC, Optical tweezers...)
- 2.0 quantum revolution $\implies$ quantum computation

References

- Phillips, W. D. (1998). *Nobel Lecture: Laser cooling and trapping of neutral atoms. Reviews of Modern Physics, 70(3), 721.*
- COHEN-TANNOUDJI, Claude N. Nobel Lecture: *Manipulating atoms with photons. Reviews of Modern Physics, v. 70, n. 3, p. 707, 1998*
- Chu, S. (1998). Nobel Lecture: *The manipulation of neutral particles. Reviews of Modern Physics, 70(3), 685.*
- Anderson, M. H., Ensher, J. R., Matthews, M. R., Wieman, C. E., Cornell, E. A. (1995).*Observation of Bose-Einstein condensation in a dilute atomic vapor. science, 269(5221), 198-201.*
- Foot, C. J. (2004). *Atomic physics (Vol. 7). OUP Oxford.*
- *COURTEILLE PW, Quantum Mechanics applied to Atoms and Light, (2022).*
- *Cohen-Tannoudji, C., Diu, B., Laloe, F. (1986). Quantum Mechanics, Volume 1. Quantum Mechanics, 1, 898.*

Instituto de Física de São Carlos